



TONBRIDGE SCHOOL

Scholarship Examination 2009

MATHEMATICS II

Wednesday 29th April 2009
2.00 p.m.

Time allowed: 1 hour 30 minutes

*Answer as many questions as you can.
All the questions carry equal marks.*

*All answers must be supported by adequate explanation.
Calculators may be used in any question.*

1. If the circular bases of two hemispheres of diameter d cm are stuck onto the circular ends of a cylinder of base diameter d cm and height h cm, the surface area S cm² of the resulting round-ended solid is given by the formula $S = \pi d(d + h)$.
- (a) If $d = 5$ cm and $h = 4$ cm, find S .
- (b) If $S = 95$ cm² and $d = 2.5$ cm, find h .
- (c) If $S = 100$ cm² and the height is twice the base diameter, find d .
- (d) What does the formula become when $h = 0$? Of what solid shape does this formula give the surface area?

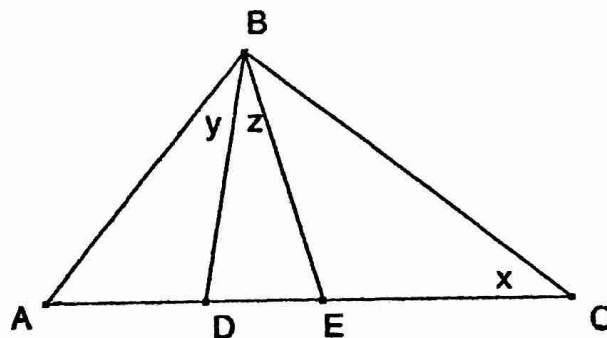
2. (a) By first adding the three equations together, or otherwise, solve the equations:

$$\begin{aligned} 3a + b + c &= 5 \\ a + 3b + c &= -2 \\ a + b + 3c &= 7 \end{aligned}$$

- (b) Solve the equations:

$$\begin{aligned} 3a + b + c + d &= 4 \\ a + 3b + c + d &= -2 \\ a + b + 3c + d &= 10 \\ a + b + c + 3d &= 0 \end{aligned} \quad \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \text{c, d}$$

3. In the figure, triangle ABC is isosceles with $AB = BC$, triangle BCD is isosceles with $BC = DC$ and triangle BEC is isosceles with $BE = EC$. Angle $BCA = x$, angle $ABD = y$ and angle $DBE = z$.
- (a) If $x = 40^\circ$, find the values of y and z .
- (b) In general, for any x , find y and z in terms of x and show that $y = z$.
- (c) Find the two possible values of x for which triangle ABE is also isosceles.

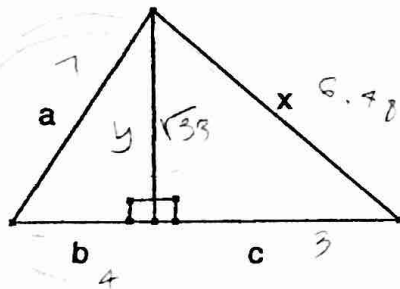


4. The following numbers are the lengths in cm of a set of 12 pencils:

10.6, 10.0, 10.1, 10.3, 2.3, 10.8, 10.6, 10.5, 10.2, 10.3, 10.7, 10.4.

- Find the median and mean lengths of the pencils.
Which is the "better" measure of average length of the pencils and why?
- What are the median and mean lengths of the pencils in inches?
(You are reminded that 1 inch = 2.54 cm.)
- Find the length of an extra pencil which, when added to the set of 12 pencils, makes the overall mean length 10.0 cm.
- How many identical new pencils, all of length 14.0 cm, must be added to the set of 12 pencils to make the overall mean length at least 13.2 cm?

5. The figure shows two right-angled triangles sharing a common side.



- If $a = 7$ cm, $b = 4$ cm, $c = 3$ cm, find x .
- Use Pythagoras's Theorem to explain why $a^2 - b^2 = x^2 - c^2$ and hence why $x^2 = a^2 + c^2 - b^2$.
- If $a = 16$ cm, $x = 10$ cm and $b = 2c$, find b .
- Give an example of four different whole numbers a, b, c, x satisfying the condition $x^2 = a^2 + c^2 - b^2$.

$$a^2 - b^2$$

$$y^2 = a^2 - b^2$$

$$c^2 + y^2 = x^2$$

$$y^2 = x^2 - c^2$$

$$100 - c^2$$

$$(a^2 - c^2)^2 (100 - c^2)$$

6. This question concerns the graph of $y = \sqrt[3]{x+100} - \sqrt[3]{x}$.

- When $x=10$ show that $y=2.64$ correct to 2 decimal places.
- Calculate the values of y for $x=0, 5, 15, 20, 25, 30$.
- Choosing sensible scales, draw a graph of $y = \sqrt[3]{x+100} - \sqrt[3]{x}$ using your values from (a) and (b).
- What happens to the graph for large, positive values of x ?
- Explain whether or not the graph makes sense for negative values of x .
- When a cube of (unknown) volume $x \text{ cm}^3$ increases in volume by 100 cm^3 , its side-length increases by 2.5 cm . Use your graph to find the value of x .

7. Study carefully the table below. Column C gives the value of the sum in Column B.

	A	B	C
Row 1	1	$\frac{0^3+1^3+2^3}{0+1+2}$	3
Row 2	4	$\frac{1^3+2^3+3^3}{1+2+3}$	6
Row 3	9	$\frac{2^3+3^3+4^3}{2+3+4}$	11
Row 4			
Row 5			
Row n			

- Write down the values that are in Columns A, B, C for Rows 4 and 5.
- What are the formulae in terms of n for the entries in Columns A, B, C for Row n ?
- Use your answer to (b) to find:
 - the value of n for which $C = 2211$;
 - the largest value of n for which the value of C does not exceed 10000;
 - an expression for $(n-1)^3 + n^3 + (n+1)^3$ without brackets and simplified as far as possible.